Derivation of Pitch Constructs from the Principles of Tone Perception

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ABSTRACT

Recent cross-cultural studies in psychoacoustics, cognitive music theory, and neuroscience of music suggest a direct correlation between the spectral content found in tones of musical instruments and the human voice on the origin and formation of musical scales. From an interdisciplinary point of view, the paper surveys important concepts that have contributed to the perception and understanding of the basic building blocks of musical harmony: intervals and scales. The theoretical model for pitch constructs derived from the perceptual attributes of musical tones - the patterns of tone intervals extracted from the harmonic series - builds on the hypothesis that fundamental assumptions of musical intervals and scales indicate physiological and psychological properties of the auditory and cognitive nervous systems. The model is based on the intrinsic hierarchy of vertical intervals and their relationships within the harmonic series. As a result, musical scales based on the perceptual and cognitive affinity of musical intervals are derived, their rapport with Western music theory suggested, and the model's potential for use in music composition implied. This leads to a vertical aspect of musical harmony by bonding of the intervallic quality and its very structure embedded within the spectra of tones that produce it. The model's application in the construction of tone systems puts forward a rich discourse between music acoustics, perception, and cognition on one end, and music theory, aesthetics, and music composition on the other.

I. INTRODUCTION

This paper examines the structure of pitched musical tones as a theoretical model for construction of vertical (harmonic) intervals and scales. Resulting from an interdisciplinary approach to Western music theory, and its historic and evolutionary relevance (Thompson, 2009; Patel 2008; Huron 2006; Christensen, 2002), the paper investigates the behavior of spectra of instrumental and vocal sounds with harmonic partials - overtones defined by integer multiples of a fundamental frequency - and pitches used to make music within the Western music tradition (Hall, 2002; Rossing et al., 2002; Benade, 1990). At the intersection of music perception, cognition, neuroscience, and music theory, the discussion unfolds around the latest studies and research in the field of music perception and cognition; its main purpose is to describe the relationships between tones defined by harmonic spectra of pitched tones and the harmonic structures pertaining to the organization of pitch material into musical intervals and scales. The paper offers an assertion according to which some novel theoretical means of explaining and constructing pitch material can be derived from the perceptual and cognitive principles of musical tones, along with their relationship with the psychoacoustical idiosyncrasies of musical timbres and related tunings (Sethares, 2004). In conclusion, the paper suggests the model's assimilation within the conventional understanding and theory of musical harmony and its compositional relevance (Temperley, 2004).

The first part of the paper begins with a review and evaluation of the literature from the auditory perception and the cognitive neuroscience of music that is related to fundamental cognitive and perceptual schemas governing our auditory processing, especially those concerning the cognition and perception of isolated musical intervals. Considering the premise of *musical harmony reflecting the very nature of musical sound* that produces it, it gives an account of important studies and research and its influence on musical grammar. Tied to the investigations of the auditory system, the survey provides a foundation upon which more complex pitch structures may be built upon (Bregman, 1990; Deutsch, 1999; Handel, 1989; Helmholtz, 1954; Hindemith, 1942; Fineberg, 2000; Krumhansl, 1990; Lerdahl, 2001; Parncutt, 1989; Patel, 2007; Peretz & Zatorre, 2003; Temperley, 2004).

The second part of the paper illustrates the formation of intervals and scales based on the assumption that the hierarchy of tones formed by the relationships within the components of the harmonic series can be interlocked into pitch class series. Founded on the described principles of interval and scale perception and cognition, I propose a theoretical framework for musical intervals and scales, along with their classification and compositional significance. Due to the model's relevance to the consideration of cross-cultural musical predispositions, I suggest that the resulting systematization of scales and modes not only forms an extension, but that it also emanates from within the traditional system of major/minor scales, modes, and artificial scales.

This results in an approach to the vertical dimension of musical harmony that is largely based on the perceptual and cognitive bases of musical tones where neurophysiological mechanisms and corresponding cognitive concepts give rise to a hierarchical organization of intervals, scales, and chords. In order to arrive at a theoretical model for the ranking of intervals and their subsequent implementation into tone systems, I hypothesize that the development of a theoretical framework for derivation of pitch scales must be based both on physical (i.e., acoustical) and psychological attributes of musical tones: it creates an interdependence between relationship and interference among physical stimuli (i.e., intervals being produced in musical context) and experience (e.g., what kind of a musical awareness or insight one might have after listening to a particular interval or chord, or a series of the same). As a result, I suggest a notion of perceptual criteria as main arbiters in the formation of the pitch material that offers a cogent impetus for its further exploration and implementation by music theorists, scientists, and composers alike.

II. PERCEPTUAL AFFINITY OF MUSICAL TONES

"Music is built of sound. Therefore we should not be surprised if its perceptual organization is governed, at least in part, by the primitive [auditory] scene-analyzing principles" (Bregman, 1990). More than a reminder of the importance and role of auditory perception in music, these lines invite us to consider the possibilities for musical syntax that may be latent within and among the sonic constituents of a musical sound. It is in this vein that I reconsider the perceptual principles that may be important in the creation of pitch constructs, schemas, and processes found in the domain of musical harmony.¹

It is well known that the perception of musical tones is conditioned by the differences in the shape of their acoustic waveforms (Moore, 2003; Angus & Howard, 2001; Roederer, 1994; Pickles, 1988; Pierce, 1983). The variations of acoustic pressure produced by the physical source of a musical instrument that impinge on the listener's tympanic membrane result in a pattern of vibration set up on the basilar membrane, whereby the peculiarity of spectral distribution define a distinct and unique timbre (Sandell, 1991). As a result, the sensation of a pitch of the most salient fundamental frequency conveys not only its most perceptible tone frequency but also its very tone color - the timbre that formed the resulting tone or what spectrum the perceived note is derived from (e.g., a tone of a note played on the flute would emit the fundamental pitch characterized by a *filtered* harmonic spectrum, and the spectrum of an inharmonic percussion instrument would elicit a virtual fundamental pitch that would be perceived as mirroring the inharmonic quality of a bell-like tone.) In this way, the perception of pitches, on the high-low continuum, is defined by a particular frequency (measured in Hertz), and with a precisely assigned pitch-name corresponding to its most salient frequency component. Because of the experimental nature of this study, its practical repercussions, and mainly its significance in the realm of Western compositional idiom, it seemed most valid to consider musical sounds that display a broad harmonic spectral energy distribution in their tones, such as those found in the most common pitched instruments of a modern orchestra (Hall, 2002; Rossing et al., 2002; Benade, 1990).

While concepts such as the role of 'virtual fundamental' (Terhardt, 1974), 'pitch salience' (Krishnan, Bidelman & Gandour, 2010; Reichweger & Parncutt, 2009; Parncutt, 2005; Hofmann-Engl, 1999; Houtsma, 1984; Terhardt, Stoll & Seewann, 1982), and 'roughness' (Vassilakis, 2001; Terhardt, 1974) hold an importance in the discussion of the nature and origin of musical harmony, it is the interrelationship between pitches and their corresponding attributes of consonance and dissonance that serve as impetus for this study. As a result of the physiological basis of consonance - fusion and neural coding of pitch relationships (Tramo et al. 2003; Gerson & Goldstein, 1978; Terhardt, 1974) and neural-firing coincidence of spectral components (Moore, 1989; Patterson, 1986; Roederer, 1973), it is the concept of harmonic, or overtone series that will be investigated as not only an extension of the Pythagorean interval ratio theory but as cognitive and perceptual motive for the estimation of intervals and classification of musical scales.

The harmonic series remains a physical entity common to all pitched tones as well as the human voice. Its significance in pitch discrimination suggests a psychological importance in the appreciation of musical harmony as both *cause* and *effect* of the perceptual importance of pitches in musical structures (Parncutt, 1989). Most importantly, the cognitive capacity of the human auditory system to process the sensory affinity of musical intervals is directly related to the perception of musical timbre its relationship with the harmonic spectrum (Sethares, 2004). In light of these observations, it is the possibility of implicit auditory and musical predispositions related to the harmonic series and its role in music apperception that forms and furthers this hypothesis for a theoretical framework of pitch schemas and their capacity as both theoretical and compositional archetypes.

A. Consonance and Dissonance

In Greek mythology, *Harmonia* was the goddess of human and divine harmony. She was the daughter of Ares, god of war, and Aphrodite, goddess of love and beauty. Harmonia has often been considered to represent the two opposite poles of energy: good and bad, positive and negative, yin and yang, and conceivably, musical *consonance* and *dissonance*. Her importance in the arts and culture could also explain the epistemological usage of the word 'harmony', which in music theory is generally understood as simultaneous soundings of different tones and the way they relate to each other (Kostka & Payne, 2004; Piston 1941/1987; Schoenberg 1922/1978).

As the basic building blocks of music, consonant and dissonant pitch relationships represent the acoustical, theoretical, psychological, and philosophical bedrock of musical harmony. There have been many ways of measuring and classifying consonant and dissonant intervals throughout music history. Over the last few centuries, several models to measure and rank intervals according to their degree of consonance and dissonance have been proposed (Tenney, 1988). What follows is an outline of the most influential empirical models of consonance and dissonance estimation; its main purpose is to provide the background for the theoretical model for construction of pitch scales from the principles of tone perception.

For a long time, music theorists have claimed perfect-ratio intervals accountable for the foundation of Western harmonic language. Combinations of particular pitches derived through the Pythagorean division of a string greatly influenced the evolution of musical harmony, by which the concept of interval ratio has become an incentive for the understanding of Western music theory up to the present day (Christensen, 2002). The Pythagoras's conception of interval ratios also conditioned Plato's view on harmony, and it directly contributed to the speculative theories for the understanding of music and psyche. For example, Aristoxenus of Tarentum (4th century BC) and Ptolemy (2nd century AD) viewed a classification of intervals as having a more sensory impact on harmony then mere numerical properties. Subsequently, in the 16th century, Vincenzo Galilei opposed any previously established numerical hierarchy of consonances found in many of Zarlino's writings. He claimed that there was a certain continuum of consonances, and in this regard he

¹ In this paper, the words *tone* and *note* refer to cognitive auditory percepts of musical *pitch* as a psychological and musical construct: tone to a label given by researchers and scientists; *note* to a graphical representation of a musical sound employed by musicians.

anticipated not only Descartes and Marsenne's empirical ideas in music theory, but also some psychoacoustical methods of classification of musical intervals (Kreitner 2011; Paterson, 2011; Palisca, 1985).

At the dawn of modern psychoacoustics, Helmholtz wrote about the sensation of consonance and dissonance as depending both on the physiological structure of the auditory system as well as on the cultural and social preferences of one's listening experience and predispositions; he also argued that the perceived roughness in an interval or chord may in turn determine its consonance or dissonance level (Helmholtz, 1877/1954). Similarly to Helmholtz, von Békésy sought to determine the cause and effect of musical consonance and dissonance while associating a relative sensation of roughness (Waver, 1960). While Plomp and Levelt later expanded on this by introducing a notion of critical bandwidth as a method of estimating the dissonance level in pairs of pure tones (Plomp and Reiner, 1965), Hutchinson and Knopoff elaborated on this by applying Plomp and Levelt's finding to musical chords (Hutchinson & Knopoff, 1978).

In his treatise on tone psychology, Stumpf raised questions as to how we determine what the actual origins and nature of music sound are, as well as to their relation to consonance, harmony, and scales (Butler and Green, 2002; Stumpf 1883-90). It was Stumpf who alluded to the unifying concept of intervallic sonance by formulating his ideas of sound fusion that went further to explain the phenomenon of consonance not so much as 'pattern matching' of the individual spectral components in the interval, but as the degree of fusion - asynchronous quality of two tones when sounded at the same time: the more an interval or a chord sounds as a single perceptual entity, the more consonant, or perceptually fused it is. More recently, the general consensus has established psychological and musicological evidence to endorse Terhardt's model of sensory consonance, which among other things suggests the absence of friction, interference, or roughness as a prerequisite for the harmonicity of consonance (Terhardt, 1984).

This notwithstanding, the real challenge of how to find a uniform way to classify musical intervals persisted. While both the music theorists and scientists have managed to place musical intervals into more or less defined categories of consonance and dissonance, the hierarchical ranking of intervals has continued to be a part of the speculative tradition of Western music theory. Arguably, this may be due to cultural predispositions, or more plausibly, due to certain perceptual and cognitive constraints associated with human auditory processing. Hence, the dichotomy of consonance and dissonance continues to present an intriguing query. How is it possible that from the listener's perspective, musical scales lead to intervals and chords that often fuse into a single perceptual unity with consonant as opposed to dissonant quality? Is it possible that when this happens, sonority's distinct character could be apprehended as its timbre in terms of a single spectral fusion or alignment, rather than a combination of discrete pitch components? What makes this query simple but highly suggestive is the cognitive importance and perceptual relevance of the sonic structure of musical sound, which directly relates to the interdependence of conscious intellectual activity of music cognition to the perceptual attributes of immediate sensory experience. It is on

the premise of these two psychological constituents of the human auditory system and their connection with the physical world of sound, its production and reception that has inspired this investigation into the cognitive origin of intervals and scales. Because of their effects on the human mind and behavior, their emotional ramifications remain to fascinate music theorists, scientists, and composers. However, within the Western music tradition, at least, and especially in contemporary music composition and research, the notion of relative consonance/dissonance has remained ambiguous at times. This equivocation may be due to the large number of conducted studies as well as a number of aesthetic trends and compositional idioms that perhaps sought to suppress the sensory aspect of musical harmony in some twentieth-century music (e.g., certain types of serial or aleatoric music).

In order to reconcile these diverse, but mutually inclusive, views, I will adopt the term *sonance* to encompass the perceptual dichotomy of consonant and dissonant intervals under a single term. Going back to Giovanni Battista Benedetti, an Italian Renaissance mathematician and physicist, sonance can be best described as relative consonance and/or dissonance of a musical interval – a continuum of pitches encompassing consonance on one end, and dissonance on the other (Palisca, 1973). Therefore, in this context, 'intervallic sonance' becomes a parameter of different degrees between two opposite poles on a consonance/dissonance continuum: the increment of sonance in an interval or in a chord results in a potentially higher or stronger consonance (e.g., the unison of two pitches would designate the highest sonance).

As Stumpf speculated on the fusion of a chord into a psychological entity, I propose to consolidate many speculations on tone sonance as a significant cognitive constituent of musical intervals, chords, and harmony. This is primarily due to the ability of our auditory system to respond to the relationships of pitches and their spectral content when forming auditory estimations of consonance and dissonance. As a result, various degrees of intervallic/harmonic sonance can then be understood as being proportional to quantitative consonance and dissonance, or harmonicity and inharmonicity. For instance, one may imagine a harmonic syntax created by bonding intervals in various ways, e.g., by ranking of intervals in regard to their inherent degrees of sonance or by creating progressive, regressive, static, or erratic structural processes based on the intervals' qualities (Berry, 1987). This notion becomes especially interesting in the domain of musical composition, where it may offer a vast array of pitch possibilities in structuring intervallic and harmonic sequences.

The most debated consonance/dissonance postulation of harmonic intervals claims that two pure or complex tones sounded together depends upon their frequency difference rather than on their frequency ratio, with ratios of lower simple numbers being more consonant than those that are higher (Rossing et al., 2002; Cope, 1997; Parncutt, 1989). The actual cause of dissonance in the ear – in terms of the physical properties of the amplitude fluctuation of individual signals – is associated with the occurrence of *interference*. As a result, various combination and difference tones originate from the mechanically nonlinear responses in the eardrum. Compared to the beating of combination and difference tones, the auditory sensation of roughness is more often explained as a reciprocal action and influence within the auditory filter on

the basilar membrane. This filter, known as *critical band* acts as a window within which certain interactions of individual spectral components are *unpleasantly* experienced as roughness. The critical bandwidth postulation on consonance and dissonance is then a relationship between perceived sonance of tones, their action on the basilar membrane, and apperception within the auditory cortex.

With the birth of cognitive neuroscience, some new and intriguing perspectives have become the subjects of debate. Most recently, music research has seen a large number of studies on aspects of musical harmony from the neurophysiological perspective. The findings indicate that there is a high correlation between tonal consonance of musical intervals and the neural activity in the auditory system. Most specifically, the studies reveal the fine timings of auditory nerve fibers activation, a reaction that is described as consisting of periodic spikes of frequency components that corresponds to related pitches from the very interval as well as to the interval's sum and difference tones (Tramo et al., 2003; Cariani et al., 1992; Cariani & Delgutte, 1992). Furthermore, the relevant studies claim that a majority of spectral frequency components for dissonant intervals cannot be resolved by central auditory neurons, causing the frequency interference among the neighboring partials, and consequently, the perception of auditory roughness. The studies also establish the notion that beating patterns cover the full range of the auditory nerves for dissonant intervals such as a minor second or a tritone; the most consonant intervals, such as a perfect fifth or a fourth induced none to almost negligible irregularities of beating patterns of auditory nerve fibers (Bharucha, 2001; Weinberger, 1999). Thus, the consonant, or highly sonant intervals, exhibit a strong periodicity and consequently registered regular beating patterns in our nervous system, contributing to the universal claim for the perception of certain intervals as concordant and others as relatively discordant musical intervals. This approach to the concept of consonance-dissonance suggests that perceptual dichotomy is mainly a function of the pitch relationships among the notes present in the interval (Tramo et al., 2003). Owing to this research, the neural coding of pitch relationships offers empirical support for the theoretical model of musical scales developed below. In the ensuing portion of the paper, I present postulatory theoretical evidence to support my contentions that (1) consonance and dissonance are emerging properties of pitch relationships dependent on their spectrum and tonal quality (2) the acoustical construct of harmonic series emanates a hierarchy of vertical intervals and (3) the perceptual hierarchy of intervals may be considered as a building agent for musical scales.

III. TONE SYSTEMS

The palette of musical intervals and their combinations comprise a wide range of percepts – from the perfectly harmonious to very discordant ones. Today, psychological correlations between consonance and dissonance convey polarities, such as pleasantness or unpleasantness, stability or instability, and in psychoacoustical terms, smoothness or roughness, euphoniousness or cacophonousness (Van de Geer et al., 1962). In the traditional study of counterpoint, for instance, the dissonant intervals are generally treated as a negative phenomenon, with a necessity for resolution into a consonant interval (Kennan, 1998; Jeppesen, 1932). Despite an abundance of aesthetic judgments on the understanding and role of consonance and dissonance in musical structures, this paper considers the apparent diversity in sonance estimation to be important for analysis and development of musical material informed by cognitive music theory.

For this reason, the cognitive attributes of consonance and dissonance offer the opportunity for exploring biological and evolutionary margins of human auditory perception as to determine why certain cognitive traits may have conditioned the way pitches are integrated into intervals, scales, and chords. Revisiting the hypothesis that there are physiological and psychological properties of the auditory and cognitive systems that may determine why some intervals are prone to being perceived as harmonious and others not, the remaining discussion expands on the construction of pitch schemas whose structural organization is deduced from the psychoacoustical understanding of harmonic series and the hierarchy of intervallic content found within. The result of the above investigation leads to a model for the estimation of vertical intervals and to the formulation of pitch scales and chords.

B. Intervals

A specific set of pitches used in a society may be very unique for that culture. As a result, a predisposition for certain pitch collections is largely socially and culturally conditioned. For instance, musical intervals may vary in size, complexity, and density: from simple disposition on pitches to clusters, with an infinitude of nuanced combinations. Some authors have even suggested terms such as *valency* or *tint* in order to describe both melodic and harmonic quality and relationships between musical intervals (Thomson, 1999). At the same time, a body of research in the field of music perception and cognition has convincingly demonstrated that there are some commonalities of preferences for similar perceptual and cognate schemas for processing of pitch combinations across different cultures (Balkwill & Thompson, 1999; Castellano et al., 1984; Kessler et al., 1984).

Taking the empirical evidence into consideration and applying it to the harmonic series and its psychoacoustical profile, I propose a heuristic model for sonance interval estimation based on the structure of a harmonic series - a ranking of vertical intervals created on the scale of most sonant (consonant) to least sonant (dissonant). It is a common argument that the given timbre of instruments ought to be applicable to a particular tuning, and in turn to a tone system that would serve as a mediator between them (Sethares, 2004). The model examines the tone intervals of *the most equally* tempered pitches within the first sixteen components of the harmonic series (Figure 1). The numbers above a harmonic indicate the amount of cents that a particular harmonic deviates from tempered tuning: -/+ 9-12 cents was taken as a tolerance factor for the 12-tone equal temperament approximation (Moore et al., 1985; Vos, 1986, 1982). Hence, for the practical reasons of dealing with the 12-tone equal temperament and its prevalence in the Western musical idiom, only the tones closely approximating the pitches of the Western twelve-tone scale are taken into account (i.e., the frequencies from the harmonic series that come closest to the

equal temperament, and that are hereby approximated to the same temperament tuning). In spite of this limit, other models employing some other tunings and more pitches per octave are also possible; the model may be also applicable to the Eastern musical traditions of musical cultures known for their idiosyncratic tunings, such as the gamelan ensembles of Indonesia.

By unlocking each consecutive interval pair of the selected tones within the ascending harmonic series, and by an examination of all available pitches, one arrives at a spatial hierarchy of the complete set of vertical intervals (Figure 1 explicates this procedure if applied to note C and illustrated in modern music notation.). This procedure entails a systematic, bottom-up proliferation of interval-pairs located in the ascending order of tones within the harmonic series. Fundamental pitch is considered perceptually the most salient, followed by the octave, which is regarded as the most consonant interval after the unison, or a prime interval. The third partial (or second overtone) generates the perfect fifth followed by the third reiteration of the fundamental pitch, which combined with the second overtone (G), forms the perfect fifth. Figure 1.a further illustrates this principle by applying the generating procedure to other intervals.

Thus, in order from the most consonant to the most dissonant, the thirteen intervals are classified hierarchically as superparticular ratios within the first seventeen components of the harmonic series, selected by the ratios of frequency pairs that come closest to the pitches from the equal temperament: *Unison* – the fundamental, *Perfect Octave* – 2:1, *Perfect Fifth* – 3:2, *Perfect Fourth* – 4:3, *Major Third* – 5:4, *Major Sixth* – 5:3, *Minor Sixth* – 8:5, *Minor Third* – 6:5, *Major Second* – 9:8, *Minor Seventh* – 9:4, *Major Seventh* – 15:8, *Minor Second* – 16:15, and *Tritone* – 17:12 (Figure 1.b).



Figure 1. Vertical interval ranking: a) intervals represented within harmonic series, b) hierarchical representation of thirteen vertical intervals

In support of the present study, it has been determined that Western listeners across age groups better discriminate interval changes in the contextual relationship of small-integer ratios, e.g., 2:1 – octave and 3:2 – perfect fifth, than of large-integer ratios such as the 17:12 – tritone (Trehub, 2003; Schellenburg & Trehub, 1996, 1994). The facility by which the consonant intervals are encoded in the human auditory system may have contributed to their structural importance in a large number of musical systems (Meyer, 1956, Sachs, 1943). The evidence of genetic parallels of musical pitch processing and musical predispositions, not only in adults immersed in a particular culture but also in infants before they have formed ties with the society, may further attest to possible universals of certain pitch combinations (Trehub, 2003; Drayna et al., 2001; Cross, 2001).

To summarize, the hierarchical representation of the vertical harmonic intervals indicates that the most consonant intervals, such as the perfect octave, perfect fifth and fourth, and major third and sixth, are found on the bottom end of the harmonic series represented by small integer ratios. On the contrary, more dissonant intervals, such as the major seventh, minor second, and the tritone are easily identified in the higher end of the harmonic series, mainly above the eighth partial and in the third octave of the overtone series. The above dissonance curve of the vertical intervals summarized in Figure 2 closely corresponds to not only the vast majority of the empirical models for consonance and dissonance models discussed earlier, but also to the common understanding of musical intervals in Western music theory. The consonance-dissonance affinity in this model is then primarily a result of the overtone composition of the harmonic series; it fluctuates as a function of both the spectrum and pitch relationships formed within, while relating the degree of sonance to particular areas of the harmonic series (i.e., consonant intervals formed in the lower octaves become proportionally more dissonant in the upper range of the harmonic series).



Figure 2. Harmonic Interval Sonance Estimation (sonance level inversely proportional to the ranking number)

C. Series, Scales and Modes

A vast majority of musical scales are constructed by interlocking discrete intervals in a series of ascending or descending strings of notes. As a result, the sonic affinity of incorporated intervals thereof may evoke a tonal ethos that often corresponds to the relative degree of consonance and dissonance found within all possible permutations of available notes in a particular scale. For that reason, musical scales and chords have been traditionally defined as combinations of discrete tones or in other words, as the superposition of interval classes (Kostka & Payne, 2003; Laitz, 2011; Aldwell & Schachter, 1989). It is my assumption that by imposing the sonance affinity of isolated vertical intervals upon a collection of pitches in a systematic manner, one may be able to create novel and structurally rich combinations of notes.

After establishing a sonance ranking of vertical intervals from the most consonant to the most dissonant, the next step in the creation of musical scales is to apply a hierarchical ordering of intervals to the construction of pitch series: the palette of structurally defined vertical intervals will serve as the material from which unordered pitch class sets may be derived. Figure 3 lists Sonance Pitch Series that are created by implementing the hierarchy of the interval rankings by which each new pitch series introduces another, more dissonant pitch to the set; there are 12 series possible, ranking from an interval of the octave to the complete chromatic scale comprising all twelve pitch classes. This suggests that an isolated interval ranking, defined by its place within the harmonic series, becomes an intervallic kernel by which new pitches are generated in each consecutive series. Therefore, the vertical intervals ranked by their degree of sonance represent the fundamental building blocks for the construction of pitch collections (Figure 3).

$$1) \stackrel{\circ}{\leftarrow} \stackrel{\circ}{\leftarrow} \stackrel{\circ}{=} \\ 2) \stackrel{\circ}{\leftarrow} \stackrel{\circ}{\leftarrow} \stackrel{\circ}{=} \\ 3) \stackrel{\circ}{\leftarrow} \stackrel{\circ}{\leftarrow} \stackrel{\circ}{=} \\ 4) \stackrel{\circ}{\leftarrow} \stackrel{\circ}{\leftarrow} \stackrel{\circ}{=} \\ 5) \stackrel{\circ}{\leftarrow} \stackrel{\circ}{\leftarrow} \stackrel{\circ}{=} \\ 5) \stackrel{\circ}{\leftarrow} \stackrel{\circ}{\leftarrow} \stackrel{\circ}{=} \\ 6) \stackrel{\circ}{\leftarrow} \stackrel{\circ}{\leftarrow} \stackrel{\circ}{=} \\ 6) \stackrel{\circ}{\leftarrow} \stackrel{\circ}{\leftarrow} \stackrel{\circ}{=} \\ 6) \stackrel{\circ}{\leftarrow} \stackrel{\circ}{\leftarrow}$$

Figure 3. Sonance Pitch Series as derived from the order of interval estimation from Figure 1 and 2

40 00 40 0

If one considers these unordered pitch class sets as being composed of interlocking intervals, the notion of scales developed in the present study does not differ much from classical writers and contemporary theorists who describe a musical scale as the combining of consonant and dissonant intervals within an octave (Seathers, 2003; Cohen, 2002; Burns, 1999; Cope, 1997). Hence, similarly to the pitch series described earlier, the pitch scales may represent structures of pitch classes sets whose formation evolved from the interlocking of hierarchically ordered intervals within an octave.

Presupposing the perceptual and cognitive constraints of the auditory system, in this paper I propose to assent to the simultaneous combining of notes to produce sonorities comprising of intervals and their theoretical, perceptual, and cognitive constructs (Dahlhaus et al., 2001). By the same token, I offer an assertion that musical scales may also be explained by the idea of intervallic interference by which scales and resulting chords act as artificial resonance that can be manipulated by any sound or interval (Fabbi, 1998).

In studies on musical scales and perception of melodic structures, it has been discovered that while different and varied, the scales across cultures display some significant perceptual constraints and similarities. Although in theory each of the thirteen series described above could represent a scale, it is the interval of a third between successive pitches and the number of elements therein that determines which series can be regarded as perceptually most salient for further theoretical and cognitive purposes. In other words, the frequency difference between each successive note in all constructed scales below is not larger than the interval of a third – a size of the critical bands from the most salient region of pitch weight (Huron, 2001). Perhaps the most relevant perceptual and cognitive universal for the present model is that the interval of an octave is commonly divided into more or less five to seven pitches (Burns, 1999) and the inequality of scale step distribution among successive pitches in a scale is a feature of a majority scales examined cross-culturally (Butler, 1989; Sloboda, 1985; Shepard, 1982). While listeners can *distinguish* a large number of pitches within an octave, due to the processing constraints of our auditory system, the number of *perceived* (processed) pitches in a musical scale is between 5 and 9, inclusive (Burns, 1999; Kobbenbring, 2004; Miller, 1956). Furthermore, the presence of a perfect fifth is directly linked with ease in interval discrimination; lack of its role in scalar constructs is associated with failure in interval discernment (Trainor & Therub, 1993; Lynch et al., 1990; Trehub et al., 1990; Cohen et al., 1987).



Figure 4. "Filtered" harmonic series as a genesis for Sonance Scales derivation

Figure 4 illustrates a reductive process within the harmonic series whose structure is important for creation of Sonance Scales. A bottom-up procedure, starting with the first 17 components of the harmonic series, it employs a hierarchical process of *element-proliferation* (Figure 4.1-3). The most equally tempered notes are extracted from the ascending overtone series to form the groups of notes to which additional pitches are systematically added; they are considered to be *explicit* because their tones are actually *present* in the harmonic series (Figure 4.2). To this basic set, other – more *dissonant* – pitches may be gradually added, starting with the major sixth and ending with the tritone; in turn, they are considered to be *implicit* because the intervals they represent are conditioned solely by pairs of the notes actually present in the harmonic series (Figure 4.3).

What can be inferred from the above derivation of pitches and their inherent psychoacoustic hierarchy is that clearly the unison, octave, and fifth are regarded as the most pitch-salient pillars of the interval perception. Other musical intervals, except the most dissonant ones, such as the minor second and tritone, do not fall into unequivocal ranking. As suggested earlier, one explanation may suggest that lower sonance rankings for other intervals beyond major thirds and perfect fourths could be attributed to a difficulty of our auditory system to resolve spectral components found in more of the intervals with more complex integer ratios. Despite this, it is a common understanding that high interval sonance outlines the very first spectral components in a musical timbre, e.g., if based on a fundamental C: C-E-G (Parncutt, 1989). In addition to supporting the common practice harmonic syntax in which a triad represents the basic building block, what type of other harmonic organization could be deduced from this perceptual principle?

In his treatise on music, De institutioni harmonische, Zarlino introduced the concept of a triad as a unit-sonority used as a building block in composition (Palisca, 1983). While largely criticized and disputed by the scientists of his time, Jean-Philippe Rameau also described in his writings on harmony and music that the harmonic intervals must be understood in the context of chords and in relation to a triad as a stable chordal point of reference. He later acknowledged the chord based on the first five components of the harmonic series - the major triad, as a fundamental building block in musical harmony (Rameau, 1722). Hugo Riemann, one of the major music theorists and musicologists of the second part of the nineteenth century, considered Rameau's writings on musical harmony in relation to sound crucial in delineating some of his own theoretical concepts. Within the unity of a triad, he interpreted the chordal dissonance as an impediment to the accord of *klang* structure (Riemann, 1893; Klumpenhouwer, 2002). In the present model for derivation of musical scales from the importance and nature of the cognitive intervallic interference, a theoretical structural construct that may resemble a chord (Figure 4.2) indicates a perceptual unit within a larger network of structure, a sonority that is comprised of many autonomous, but as discussed earlier, interdependent entities (Dahlhaus, 1990).

As with the interval ranking, some tones within the harmonic series stand out more than others. We easily notice that here, too, a certain cluster of pitches can be filtered out from the harmonic series: one can trace the five most-equally-tempered tones of the harmonic series, a pentachord C-G-E-D-B. This chord-sonority is comprised of the *explicit* interval-pitches derived from within the first three octaves of the harmonic series (Figure 4.2); it consists of a perfect octave, perfect fifth, major third, major second, and major seventh, all erected on the center pitch of C; its purpose here is to serve as a basic nucleus in creating pitch scales based on the perceptual order of musical intervals.

Figure 5 provides an inventory of all five- to nine-pitch scales that were generated using the above procedure. It depicts three groups of scales, all constructed upon the base collection of pitches described above and illustrated in Figure 4.2, and with increasing levels of dissonance found in the intervallic content within. Scale groups 2 and 3 comprise additional subgroups, the result of both an increase of pitch elements and dissonance levels. As it can be seen, the relationship of the intervallic interference embedded within the very consonant, fundamental pitch class set obtained from the very bottom of the harmonic series. Hence,

a Sonance Scale is understood as a collection of pitch classes or a tone family of notes directly linked to the hierarchical synopsis of the musical intervals implemented in the Sonance Series.

The constructed Sonance Scales suggest progressions in sonance affinity from very consonant scalar constructs to more dissonant ones, with the dissonance proportionate to the note density. In this regard, these *overtone* scales correspond to dissonance curves derived from the harmonic spectrum, an attribute that may suggest convincing progressive, regressive, erratic, and static structural actions, as well as tonal processes of prolongation and progression (Sethares, 2004; Lerdahl, 2001; Berry, 1976)



Figure 5. Sonance Scales



Figure 5. Sonance Scales (cont.)

Similar to the construction of the pentatonic, plainchant, and Indian modes, Sonance Scales can be further expanded to denote corresponding modes. In other words, each scale can be extended to form modes, each with its own tonal center or pitch centricity.² In this instance, the modes operate according to the intervallic distance of each tone from a designated center pitch, resulting in a predefined tonal hierarchy between individual components of the mode. The set of available modes acts in the same way as the Greek scalar genera: the modes serve to exert a particular harmonic color to a musical structure (i.e., fundamental or *plagal* modes correspond to the scalar constructs on each successive pitch of an original, or *authentic* mode).

IV. CONCLUSION

One may consider pitch combinations as a function of musical sound with a myriad of complex sonic sensations. Likewise, musical harmony may be conceived as comprising structural and emotional integration of the intervals that emerge out of the spectrum of a musical instrument. As suggested earlier, musical harmony is a perceptual and cognitive wonder that dwells in any resonance, including our own voice. This is indeed a fascinating idea, especially because of its potential to convey a vast range of impressions apt to be explored compositionally. The composer, performer, and listener are given an opportunity to imagine and experience abundant aural impressions and depict them musically. In order to achieve this, one may theoretically imagine different intervallic and harmonic degrees commensurate to their relative degree of consonance and dissonance, an attempt that may reconcile particular tone spectra of musical sounds and the resultant psychological states in music compositions (Parncutt & Hair, 2011).

The investigation on the nature of musical intervals and the resulting expansion of the existing musical scales offer a compelling approach for generating new harmonic structures: from very simple ones such as dyads to more complex chords and harmonic progressions. In turn, those renditions may be portrayed in sound as transformations of melodic lines into chords and harmonies. From the composer's perspective, the approach to formation of harmonic structures derived from the perception of musical tones puts forward an inspiring new way to view musical harmony as part of cognitive music theory. The resulting attitude towards musical composition not only brings forth some vibrant harmonic structures, but most importantly, it promotes and reflects the nature of musical harmony mirrored in the very sound that produces it.

Still, whether there is an ideal way to define, measure, and rank consonance and dissonance of musical intervals remains a debate. In most cases, relative consonance and dissonance of musical intervals may be culturally conditioned. In spite of this, there is always a place for investigation as to determine whether there might be firm perceptual or cognitive evidence behind this attribute of tone perception. From the composer's perspective, what is most fascinating about the intervallic and scalar constructs developed in this paper is the generative nature of the resulting harmonic system and its sensory affinity, as well as its harmonic structure based on strong cognitive overtones.

The scales derived in the present study are created under the presumption that they resonate well within particular timbres, and may not sound good in some other, alternative tunings for which different models for pitch constructs might need to be created (Sethares, 2004). Many questions remain unanswered, such as whether the model would enable one to organize harmony progressions according to sonance degrees or levels of intervals and chords, or whether introducing sonance curves may enhance other structural functions within a piece of music (Berry, 1976). Assuming that the perception of musical intervals and scales may vary with distances in pitch space, could tone systems be designed as to encompass the cognitive hierarchy of pitch space (Lerdahl, 2001)? Could one derive an additional system of scales based on micro intervallic tunings and what procedure, similarity, and musical significance would it have in comparison to the present study? Considering that the intervallic sonance may vary with the pitch register and musical timbre, could tone systems be designed to encompass both of these perceptual attributes? How would this sonance-based system adhere to a multidimensionality of timbre perception? This may suggest a sensory affinity of this newly created musical vocabulary,

² In this study, the concept of a 'mode' describes groups of pitch class complexes, not different from the major-minor system of the common practice period, as well as examples of artificial scales, e.g., whole-tone or Messiaen's *Modes of Limited Transpositions*.

and its structural functions in the domain of harmony and tonality.

The most significant impetus for this study is the premise that many types of music all over the world and throughout the ages are conditioned by some basic human processing predispositions – cognitive preference for consonance discrimination and musical pitch constructs based on small and unequal scale steps (Trehub, 2003). Therefore, the cognitive predispositions in human auditory processing support its perceptual validity within a traditional theoretical view of musical harmony.

Tone systems with their corresponding pitches, intervals, and chords continue to evolve as the culture of their origin does. It does so conspicuously well by not allowing itself to embrace a single musical aesthetic, idiom, or a mode of thought. Similarly to the goddess *Harmonia*, who upon her wedding was presented with a gift of a necklace and garment, musical harmony along with its intervals, scales, and chords is a musical endowment that continues to be explored by composers, performers, and listeners alike.

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REFERENCES

- Aldwell, E. & C. Schachter (1989). *Harmony and Voice Leading*. San Diego, CA: Harcourt Jovanovich.
- Angus, J. & D. M. Howard (2001). *Acoustics and Psychoacoustics*. Amsterdam and Boston: Focal Press.
- Balkwill, L. & W. F. Thompson (1999). A cross-cultural investigation of the perception of emotion in music. *Music Perception*, 17, 43-64.
- Benade, A. H. (1990). *Fundamentals of Musical Acoustics*. New York, NY: Dover Publication, Inc.
- Berry, W. (1976). *Structural Functions in Music*. New Jersey: Prentice Hall.
- Bharucha, J. J. (1991). Pitch, harmony, and neural nets: A psychological perspective. In P. Todd & G. Loy (Eds.), *Music and Connectionism* (pp. 84-99). Cambridge, MA: MIT Press.
- Bregman, A. S. (1990). Auditory Scene Analysis: The Perceptual Organization of Sound. Cambridge, MA: The MIT Press.
- Burns, E. M. (1999). Intervals, scales, and tuning. In D. Deutsch (Ed.), *The Psychology of Music* (pp. 215-264). San Diego, CA: Academic Press.
- Butler, D. (1989). Describing the perception of tonality in music: a critique of tonal hierarchy theory and proposal for a theory of intervallic rivalry. *Music Perception*, *6*, 219-242.
- Castellano, M. A., J. J. Bharucha & C. L. Krumhansl (1984). Tonal hierarchies in the music of north India. *Journal of Experimental Psychology: General*, *113*, 394-412.

- Cohen, A. J., L. A. Thorpe & S. E. Trehub (1987). Infants' perception of musical relations in short transposed sequences. *Canadian J. Psychol.*, *41*, 33-47.
- Cohen, D. E. (1993). Metaphysics, ideology, discipline: Consonance, dissonance, and foundations of Western polyphony. *Theoria*, 7, 1-85.
- Cope, D. (1997). *Techniques of the Contemporary Composer*. New York, NY: Schirmer Books.
- Christensen, T. (Ed.) (2002). *The Cambridge History of Western Music Theory*. Cambridge, UK: Cambridge University Press.
- Dahlhaus, C. (1990). Studies in the Origin of Harmonic Tonality. (R. O. Gjerdingen, Trans.) Princeton, NJ: Princeton University Press.
- Deutsch, D. (Ed.) (1982). *The Psychology of Music*. New York, NY: Academic Press.
- Dahlhaus, C. J. Anderson, C. Wilson, R. Cohn & B. Hyer. (2007, September 16). *Harmony*. Grove Music Online, L. Macy, ed. Retrieved October 5, 2001, from
- http://grovemusic.com.turing.library.northwestern.edu
- DeWitt, L. & R. Crowder (1987). Tonal fusion of consonant musical intervals: The oomph in Stumpf. *Perception and Psychophysics*, 41, 73-84.
- Drayna, D., A. Manichaikul, M. de Lange, H. Sneider & T. Spector (2001). Genetic correlates of musical pitch recognition in humans. *Science*, *291*, 1969-72.
- Fabbi, R. (1998). Theological Implications of Restrictions in Messiaen's Compositional Process. In S. Bruhn (Ed.), *Messiaen's Language of Mystical Love* (pp. 63-64). New York, NY: Garland Publishing, Inc.
- Forte, A. (1974). *Tonal Harmony in Concept and Practice*. New York, NY: Holt, Rinehart and Winston.
- Green, B. & D. Butler (2002). From acoustics to Tonpsychologie. In T. Christensen (Ed.), *The Cambridge History of Western Music Theory* (pp. 246-271). Cambridge, UK: Cambridge University Press.
- Guernsey, M. (1928). The role of consonance and dissonance in music. *American Journal of Psychology*, 40, 173-204.
- Hall, D. E. (2002). *Musical acoustics*. Pacific Grove, CA: Brooks/Cole.
- Haluska, J. (2004). *The Mathematical Theory of Tone Systems*. New York, NY: Marcel Dekker, Inc.
- Handel, S. (1989). Listening. An Introduction to the Perception of Auditory Events. Cambridge, MA: The MIT Press.
- Helmholtz, H. L. F. (1954). On the Sensations of Tone as a Physiological Basis for the Theory of Music. (A. J. Ellis, Trans.) New York, NY: Dover.
- Hindemith, P. (1942). *The Craft of Musical Composition*. New York, NY: Schott.
- Huron, D. (1991). Tonal consonance versus tonal fusion in polyphonic sonorities. *Music Perception*, 9, 1, 135-154.
- Huron, D. & P. Sellmer (1992). Critical bands and the spelling of vertical sonorities. *Music Perception*, 10, 2, 129-149.
- Huron, D. (2001). Tone and voice: a derivation of the rules of voice-leading from perceptual principles. *Music Perception*, 19, 1, 6-10.
- Jeppesen, K. (1939). Counterpoint: The Polyphonic Vocal Style of the Sixteenth Century. (G. Haydon, Trans.) Englewood Cliffs, NJ: Prentice Hall, Inc.
- Johnston, I. (2002). *Measured Tones: The Interplay of Physics and Music*. Bristol: Institute of Physics Publishing.
- Kessler, E. C., C. J. Hansen & R. N. Shepard (1984). Tonal schemata in the perception of music in Bali and in the west. *Music Perception*, *2*, 131-165.
- Kobbenbring, M. (2004). The effect of uniqueness and the number of scale tones on the detection performance for tone mistunings in scalar material. In S. D. Lipscomb at al. (Eds.), *Proceedings of the Eighth International Conference on Music Perception and Cognition in Evanston* (pp. 137-138). Adelaide, Australia: Casual Productions.

- Kostka, S. & D. Payne (2004). Tonal Harmony. With an Introduction to Twentieth-century Music. New York, NY: McGraw Hill.
- Kreitner K. (Ed.) (2011). *Renaissance music*. Burlington, VT: Ashgate Pub. Co.
- Krumhansl, C. L. (1990). Cognitive Foundations of Musical Pitch. New York, NY: Oxford University Press.
- Levitin, D. J. (2006). This is Your Brain on Music: The Science of a Human Obsession. New York, NY: A Plume Book.
- Lester, J. (2002). Rameau and eighteenth-century harmonic theory. In T. Christensen (Ed.), *The Cambridge History of Western Music Theory* (pp. 753-777). Cambridge, UK: Cambridge University Press.
- Lerdahl, F. (2001). *Tonal Pitch Space*. New York, NY: Oxford University Press.
- Lipps, T. (1995). Consonance and Dissonance in Music. (W. Thomson, Trans.) San Marino, CA: Everett Books.
- Lynch, M. P., R. E. Eilers, D. K. Oller & R. C. Urbano (1990). Innateness, experience, and music perception. *Psychological Science*, 1, 272-276.
- Meyer, L B. (1956). *Emotion and Meaning in Music*. Chicago, IL: University of Chicago Press.
- Miller, G. A. (1956). The magical number seven, plus or minus two: some limits on our capacity for processing information. *The Psychological Review*, *63*, 81-97.
- Moore, B. C. J. (2003). An Introduction to the Psychology of Hearing. New York, NY: Academic Press.
- Moore, B. C. J., R. W. Peters & B. R. Glasberg (1985). Thresholds for the detection of inharmonicity of complex tones. *Journal of the Acoustical Society of America*, 77, 1861-1867.
- Parncutt, R. (1989). Harmony: A Psychoacoustical Approach. Berlin: Springer-Verlag.
- Parncutt, R., & McPherson, G. E. (Eds.) (2002). The science and psychology of music performance: Creative strategies for teaching and learning. New York: Oxford University Press.
- Parncutt, R. & G. Hair (2011). Consonance and dissonance in music theory and psychology: disentangling dissonant dichotomies. *Journal of Interdisciplinary Music Studies*, 5, 2,119-166.
- Peretz I. & R. Zatorre (Eds.) (2003). The Cognitive Neuroscience of Music. Oxford, UK: Oxford University Press.
- Peterson M. A. (2011). *Renaissance mathematics and the arts*. Cambridge, MA: Harvard University Press.
- Pierce, J. R. (1983). The Science of Musical Sound. New York, NY: Scientific American Books, W. H. Freeman and Co.
- Pickles, J. O. (1988). An Introduction to the Physiology of Hearing. New York, NY: Academic Press.
- Piston, W. (1987). *Harmony*. (M. DeVoto, Rev.) New York: W. W. Norton & Company, Inc. (Original work published 1941.)
- Roederer, J. G. (1994). *The Physics and Psychophysics of Music*. New York, NY: Springer-Verlag.
- Rossing T. D., F. R. Moore & P. A. Wheeler (2003). *The Science of Sound*. San Francisco, CA: Addison Wesley.
- Sachs, C. (1943). *The Rise of Music in Ancient World: East and West*. New York, NY: Norton.
- Sandell, G. J. (1991). Concurrent timbres in orchestration: a perceptual study of factors determining "blend". (Unpublished doctoral dissertation). Northwestern University, Evanston, IL
- Schellenberg, E. G. & S. E. Trehub (1994). Frequency ratios and the discrimination of pure tone sequences. *Perception and Psychophysics*, 56, 472-478.
- Schellenberg, E. G. & S. E. Trehub (1996). Natural musical intervals: evidence from infant listeners. *Psychological Science*, 7, 272-277.
- Schellenberg, E. G. & S. E. Trehub (1996). Children's discrimination of melodic intervals. *Developmental Psychology*, 32, 1039-1050.
- Schneider, A. (1997). "Verschmelzung", tonal fusion, and consonance: Carl Stumpf revisited. In M. Leman (Ed.), Music, Gestalt, and Computing. Studies in Cognitive and Systematic Musicology (pp. 117-143). Berlin: Springer.

- Schoenberg, A. (1978). Theory of Harmony. (R. E. Carter, Trans.) Pacific Palisades, CA: Belmont Music Publishers (Original work published 1922).
- Shepard, R. N. (1982). Geometrical approximations to the structure of musical pitch. *Psychological Review*, 41, 489-504.
- Sloboda, J. A. (1985). The Musical Mind: The Cognitive Psychology of Music. Oxford, UK: Clarendon Press.
- Temperley, D. (2004). *The Cognition of Basic Musical Structures*. Cambridge, MA: The MIT Press.
- Teenney, J. (1988). A History of Consonance and Dissonance. New York, NY: Excelsior.
- Trainor, L. J. & S. E. Trehub (1993). Musical context effects in infants and adults: key distance. *Journal of Experimental Psychology: Human Perception and Performance*, 19, 615-26.
- Trainor, L. J. & S. E. Trehub (1993). What mediates infants' and adults' superior processing of the major over the augmented triad? *Music Perception*, 11, 185-96.
- Trehub, S. E., L. A. Thorpe & L. J. Trainor (1990). Infants' perception of good and bad melodies. *Psychomusicology*, *9*, 5-19.
- Trehub, S. E., E. G. Schellenberg & D. Hill (1997). The origins of music perception and cognition: a developmental perspective. In I. Deliège & J. Sloboda (Eds.), *Perception and Cognition of Music* (pp. 103-128). Hove, UK: Psychology Press.
- Trehub, S. E. (2003). Musical predisposition in infancy: an update. In I. Peretz & R. Zatorre (Eds.), *The Cognitive Neuroscience of Music* (pp. 3-20). Oxford, UK: Oxford University Press.
- Van de Geer, J. P., W. J. M. Levelt & R. Plomp (1962). The connotation of musical consonance. *Acta Psychologia*, 20, 308-319.
- Vos, J. (1986). Purity ratings of tempered fifths and major thirds. *Music Perception*, 3 (3), 221-258.
- Vos, J. (1982). The perception of pure and mistuned musical fifths and major thirds: thresholds for discrimination, beats, and identification. *Perception and Psychophysics*, *32*, 297-313.