Proceedings of the 12th International Conference on Music Perception and Cognition and the 8th Triennial Conference of the European Society for the Cognitive Sciences of Music, July 23-28, 2012, Thessaloniki, Greece
Cambouropoulos E., Tsougras C., Mavromatis P., Pastiadis K. (Editors)

A Directional Interval Class Representation of Chord Transitions

Emilios Cambouropoulos,

School of Music Studies, Aristotle University of Thessaloniki, Greece
emilios@mus.auth.gr

ABSTRACT

Chords are commonly represented, at a low level, as absolute pitches (or pitch classes) or, at a higher level, as chords types within a given tonal/harmonic context (e.g. roman numeral analysis). The former is too elementary, whereas, the latter, requires sophisticated harmonic analysis. Is it possible to represent chord transitions at an intermediate level that is transposition-invariant and idiom-independent (analogous to pitch intervals that represent transitions between notes)?

In this paper, a novel chord transition representation is proposed. A harmonic transition between two chords can be represented by a *Directed Interval Class (DIC) vector*. The proposed 12-dimensional vector encodes the number of occurrence of all directional interval classes (from 0 to 6 including +/- for direction) between all the pairs of notes of two successive chords. Apart from octave equivalence and interval inversion equivalence, this representation preserves directionality of intervals (up or down). Interesting properties of this representation include: easy to compute, independent of root finding, independent of key finding, incorporates voice leading qualities, preserves chord transition asymmetry (e.g. different vector for I→V and V→I), transposition invariant, independent of chord type, applicable to tonal/post-tonal/ atonal music, and, in most instances, constituent chords from a chord transition can be uniquely derived from a DIC vector. DIC vectors can be organised in different categories depending on their content, and distance between vectors can be used to calculate harmonic similarity between different music passages. Preliminary tests are presented using simple tonal chord sequences and jazz sequences.

This proposal provides a simple and potentially powerful representation of elementary harmonic relations that may have interesting applications in the domain of harmonic representation and processing.

I. INTRODUCTION

In recent years an increasing number of studies propose computational models that attempt to determine the harmonic distance between two pieces/excerpts of music, primarily for music information retrieval tasks (Allali 2007, 2010; de Haas, 2008, 2011; Hanna et al., 2009; Paiement 2005; Pickens et al. 2002). Such models assume a certain representation of chords and, then, define a similarity metric to measure the distance between chord sequences. Chords may be represented as chroma vectors (pitch class profile), or chord root transitions within a given tonality (following harmonic analysis), or, even, abstract chord distances (see below). In the case of absolute pitch representation, such as chroma vectors, transpositions are not accounted for. Allali concludes that “no pitch representation correctly enables retrieval polyphonic music systems to be transposition invariant.” (Allali et al, 2007, p.29). If harmonic analytic models are used to derive a harmonic description of pieces, more sophisticated processing is possible as chords maybe represented as degrees within keys or tonal functions and so on, but these models rely on complicated harmonic analytic systems (such as Temperley’s Melisma model - 2001).

All the above models rely on some representation of individual chords. There are a few attempts, however, to represent chord *transitions*. For instance, de Haas et al. (2008, 2011) represent chord transitions as chord distance values adapting a distance metric from Lerdahl’s *Tonal Pitch Space* (2001); however, a chord transition being represented by a single integer value seems to be an excessive abstraction that potentially misses out important information. This paper explores the possibility of a richer chord transition representation that can be readily derived directly from the musical surface and that has many interesting and useful properties.

In the first section below I will try to show the need for a new chord transition representation, which is analogous to the melodic interval representation. Then, the new Directed Interval Class (DIC) representation will be introduced and some of its potentially useful properties will be highlighted. Finally, the DIC representation will be used as the basis for two preliminary tests on a harmonic similarity task for different musical idioms. Finally, a brief discussion will summarise the importance of the proposed representation and will suggest interesting new avenues for further exploration.

II. MELODIC VS HARMONIC INTERVAL REPRESENTATION

Before discussing harmonic content, I would like to examine aspects of melodic representation. This brief discussion will make clear what shortcoming in the harmonic domain the proposed harmonic representation attempts to address. In this paper we assume a discrete 12-tone equal-tempered pitch system.

At the lowest symbolic level, a melody can be represented as a sequence of absolute pitches (e.g. MIDI pitch numbers), or as a sequence of relative pitch intervals (e.g. in semitones). These simple low-level encodings require no further idiom-specific knowledge, such as scales, keys, tonal hierarchies. Any melodic sequence (e.g. tonal, atonal, modal, post-tonal, serial, etc.) can be represented as a series of absolute pitch numbers, or can be automatically converted to a sequence of relative pitch intervals (in semitones). If octave equivalence is assumed, pc can be used for absolute pitch and pitch intervals can be represented as simple pitch intervals (without octaves).

The relative interval representation is closer to the way most listeners perceive pitch sequences (of course there exist a smaller proportion of listeners with absolute pitch – see, for instance, Levitin and Rogers 2005). The interval encoding facilitates *transposition-invariant* storage and comparison of melodic sequences (as opposed to the absolute pitch encoding). This low-level interval representation may be hypothesised to be close to the way naïve listeners (that have no implicit higher-level knowledge of a specific musical idiom) perceive a melodic sequence.

Obviously, an acculturated listener into a specific idiom (e.g. tonal idiom) uses more advanced representations to interpret and encode melodic sequences (e.g. scale degrees within a tonality). The construction of such sophisticated representations requires more complex cognitive 'processing', but, in return, facilitates more efficient and sophisticated comparisons between different melodic material allowing 'meaningful' musical entities to emerge (e.g. melodic motives, themes).

Let us now turn our attention to harmony. Let's assume, for simplicity, that we have a simple progression of chords, i.e. mere vertical sonorities (without any ornamentations). By chords we mean any vertical sonority of two or more notes (not merely tonal triads). How can chords be represented in line with the above discussion on melodic representation?

At the lowest levels, chords are simply co-sounding absolute pitches. This representation is too naïve to account for any aspect of harmony. At the level of an idiom-specific representation, and, more specifically, tonal representation, there are various harmonic descriptions, such as figured bass, traditional roman numeral analysis, guitar style chords, functional harmonic description (T/S/D), and so on. All these analytic interpretations of the actual vertical sonorities, require specialised knowledge regarding tonal centres, scales, keys, chord roots, chord root relations (circle-of-fifths) and so on. Even figured bass, the simplest and poorest of all these representations, requires scale knowledge so that the numbers above the bass note can be defined (e.g. a 3 above C is either Eb or E). Such tonal harmony analytic descriptions are anything but trivial.

The question arises: is there any harmonic representation that is idiom-independent and traspositionally invariant (relative pitch)? That is, is it possible to represent harmony at a higher level than the primitive absolute pitches and, at the same time, at a level that does not require domain specific musical knowledge? Is it possible to define a harmonic equivalent to the idiom independent transposition invariant 'pitch interval'?[1] Is it possible to devise a representation to fill in the gap in Table 1?

Table 1 Melodic and harmonic representation in relation to transposition invariance and idiom-independency.

		Melodic Representation	Harmonic Representation
Idiom indepen-dent	Transp. sensitive	Absolute pitch	Absolute Pitch
	Transp. invariant	Intervals (semitones)	???
Idiom-dependent (e.g. tonal)		Trad. Note Notation Scale degrees	Figured bass Roman Numerals Guitar style chords

[1] It should be noted that in terms of individual chords, it is very simple to represent them as a set of intervals (traditional tonal intervals or interval vectors), but in case of tonal music these interval sets are trivial (for instance, major-minor chords consist of the same intervals: inteval vector: 001110).

III. THE DIRECTED INTERVAL CLASS (DIC) CHORD TRANSITION REPRESENTATION

A novel chord transition representation is proposed. A harmonic transition between two chords can be represented as a Directional Interval Class (DIC) vector. The proposed 12-dimensional vector encodes the number of occurrence of all directional interval classes (from 0 to 6 including +/- sign for direction) between all the pairs of notes of two successive chords. That is, from each note of the first chord all intervals to all the notes of the second chord are calculated. Direction of intervals is preserved (+,-), except for the unison (0) and the tritone (6) that are undirected. Interval size takes values from 0-6 (interval class). If an interval X is greater than 6, then its complement 12-X in the opposite direction is retained (e.g. ascending minor seventh '+10' is replaced by its equivalent complement descending major second '-2').

The 12-dimensional DIC vector features the following directed interval classes in its twelve positions: 0 (unison), +1, -1, +2, -2, +3, -3, +4, -4, +5, -5, 6 (tritone). For instance, the transition vector for the progression I→V is given by the DIC vector: Q = <1,0,1,1,1,1,0,1,0,0,3,0> (which means: 1 unison, 0 ascending minor seconds, 1 descending minor second, 1 ascending major second, etc.) – see Figure 1, and further examples in Figure 2.

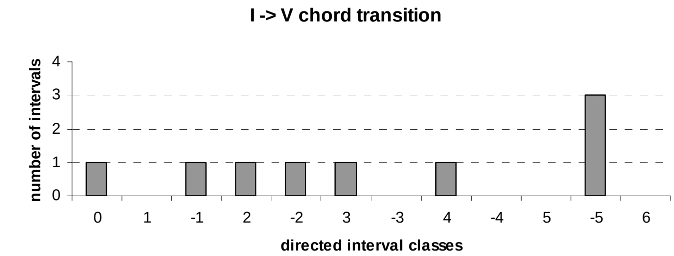

Figure 1 The DIC vector: <1,0,1,1,1,1,0,1,0,0,3,0> for the chord transition I→V depicted as a bar graph.

The DIC vector is unique for many tonal chord transitions. However, there are a number of cases where different tonal transitions have the same vector. For instance, the transitions I→V and IV→I share the same DIC vector as their directed interval content is the same; it should be noted, that, heard in isolation (without a tonal centre reference), a human listener cannot tell the difference between these two transitions.

The DIC vector uniquely determines the two chords that comprise the transition. This is true for all cases except when one of the two chords is symmetric, such as augmented chord, or diminished seventh chord. This is actually an interesting finding that agrees with music theory; for instance, diminished seventh chords are considered ambiguous and can resolve to different chords leading to different tonal regions/keys.

The proposed DIC representation preserves directionality of intervals (up or down), and, therefore, it incorporates properties of voice leading. For instance, the DIC vector naturally accommodates chord transition asymmetry. If the two chords in a chord transition are reversed, the absolute values of intervals are retained; however, the directions of intervals are reversed. This way, the vectors, for instance, for the I→V transition and the V→I transition, are different (compare, DIC vectors of Figure 1 and Figure 2a (top) - see also numerical distance between them in next section).

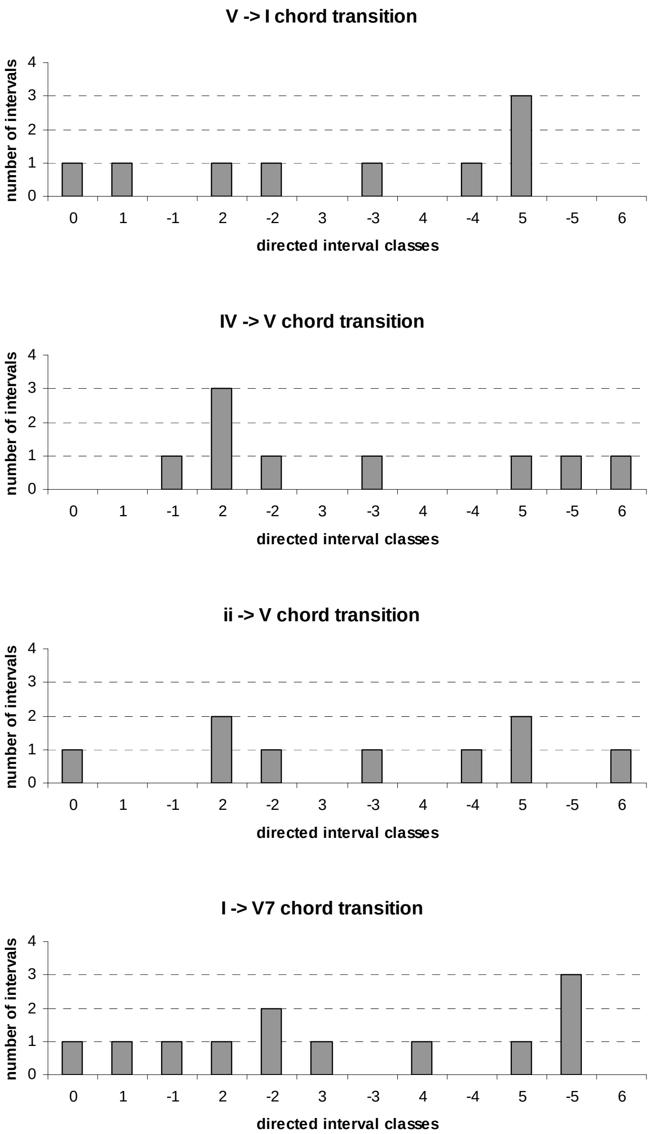

Figure 2 DIC vectors for four standard tonal chord transitions: V→I, IV→V, ii→V, I→V7

IV. HARMONIC DISTANCE VIA DIC VECTORS: TWO PRELIMINARY TESTS

In this section, we attempt to test the effectiveness of the DIC vector representation in a harmonic progression similarity task. The main assumption here is that, if this representation encodes sufficiently aspects of the harmonic content of chord progressions, then, given an appropriate distance metric, similarity between different chord progressions can be calculated and evaluated. We herein use the DIC vector representation as a basis for calculating the distance between simple chord progressions in two preliminary tests. In one case, we calculate the distances between 12 simple tonal triadic progressions, whereas in the second case, we have 11 jazz progressions.

For this preliminary testing, we employ a very simple distance metric, namely the city block distance. The city block distance between two vectors is defined as the sum of the absolute differences of their coordinates. Essentially it counts the number of intervals that are different between two vectors. The total distance between two chord progressions is the sum of the distances between their corresponding DIC vectors. There are various other more advanced ways to measure the distance between two DIC vectors, but this simple metric will suffice for this preliminary test. The city block distance between DIC vectors P and Q is:

$$d(\mathrm{P},\mathrm{Q}) = \sum_{i=1}^{n} |p_i - q_i|$$

where $\mathrm{P}=(p_1, p_2, p_3, \dots\ p_n)$ & $\mathrm{Q}=(q_1, q_2, q_3, \dots\ q_n)$.
Some examples: *d*(I→V, I→V7)=3, *d*(ii→V, IV→V)=6, *d*(I→V, IV→V)=8, *d*(I→V, V→1)=10, *d*(V→I, IV→V)=10.

The total distance between two chord progressions C_1 & C_2 is defined as the sum of the city block distances between each pair of corresponding chord transitions. For two equal-length chord sequences that consist of *m* chords, we have two corresponding sequences of *m-1* DIC vectors: $\mathrm{P}=(\mathrm{P}_1, \mathrm{P}_2, \mathrm{P}_3, \dots \mathrm{P}_{m-1})$ and $\mathrm{Q}=(\mathrm{Q}_1, \mathrm{Q}_2, \mathrm{Q}_3, \dots \mathrm{Q}_{m-1})$. The distance between the two chord progressions is calculated as:

$$d(\mathrm{C}_1,\mathrm{C}_2) = \sum_{i=1}^{m-1} d(\mathrm{P}_i,\mathrm{Q}_i)$$

Cluster analysis can be used to group objects based on a given distance matrix. In this paper we use phylogenetic trees (branching diagrams) that can be used to visualise distance or similarity relations that exist between members of a group of entities (e.g. genetic materials, or cultural objects) – for use of phylogenetic trees in relation to musical rhythms see Toussaint et al. (2011). A phylogenetic tree is constructed such that the distance in the branches corresponds as closely as possible to the corresponding distance in the distance matrix. We used the software application SplitsTree-4 (Huson, 1998) for constructing phylogenetic trees from our chord sequence distance matrices.

In a first preliminary test, we constructed a set of twelve triadic tonal chord progressions, each consisting of 4 chords (see Figure 3). A simple computer application calculates the distance between every pair of these chord progressions according to the measure described above creating a 12x12 distance matrix. From this matrix, a phylogenetic tree is constructed using the SplitsTree-4 package (see Figure 3). The phylogenetic tree splits the twelve progressions into three groups that correspond with each staff in Figure 3. If the chord sequences are examined more closely, it is clear that in terms of functional harmony the chord progressions in the first staff correspond to the progression T-S-D-T[2], in the second staff to T-S-T-D, and in the third staff to S-T D-T. This very simple method manages to group together successfully these chord progressions without any knowledge of tonality, keys, chord roots, scales or any other sophisticated harmonic concept. Even if we transpose these progressions to various keys the proposed method would give exactly the same result (this whole process is transposition invariant). The mere intevallic content of these progressions is sufficient for finding similarities between these progressions and organising them into groups. It should be noted, however, that, if in this example more extensive substitutions of chords are introduced, the resulting tree is less successful, possibly because the distance metric is extremely elementary. Further research and testing are required.

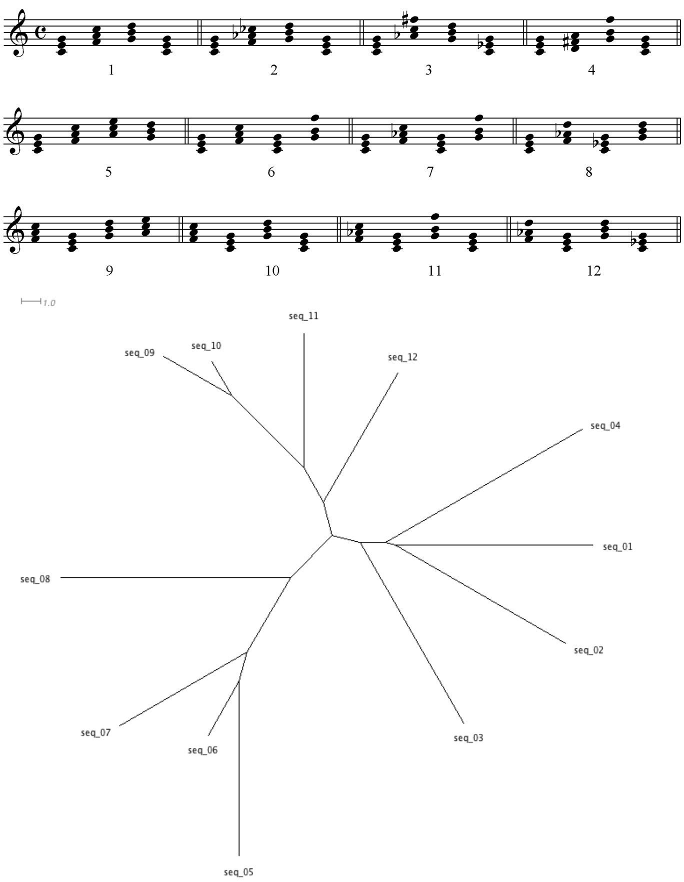

Figure 3 The triadic chord progressions are organised into a phylogenetic tree that illustrates their similarities and grouping based on their DIC vector distance.

In a second preliminary test, we asked an experience piano jazz performer to write down some jazz chord progressions (same length) and also to let us know how she thinks they relate to each other. The jazz pianist prepared eleven jazz chord progressions each consisting of 4 chords (see Figure 4). As in the previous test, these were organised into an 11x11 distance matrix and, then, a phylogenetic tree was constructed (see Figure 4). The jazz musician examined the resulting phylogenetic tree and stated: "I think it is very nice and I agree with the main parts. The point on which I would disagree is placing 6 far away from 5,7,8; I would place them in a same class. Secondly, group 1, 3 is closer to 2 in my opinion; the rest of the tree is very convincing." The harmonic similarity between these jazz chord progressions seems to be captured reasonably well, despite the simplicity of the proposed model and its total ignorance of jazz harmony. This is a positive indication. Yet, more systematic research is necessary to improve the model and to test it more extensively (e.g. empirical data for the 11 progressions could be gathered from a larger number of jazz musicians).

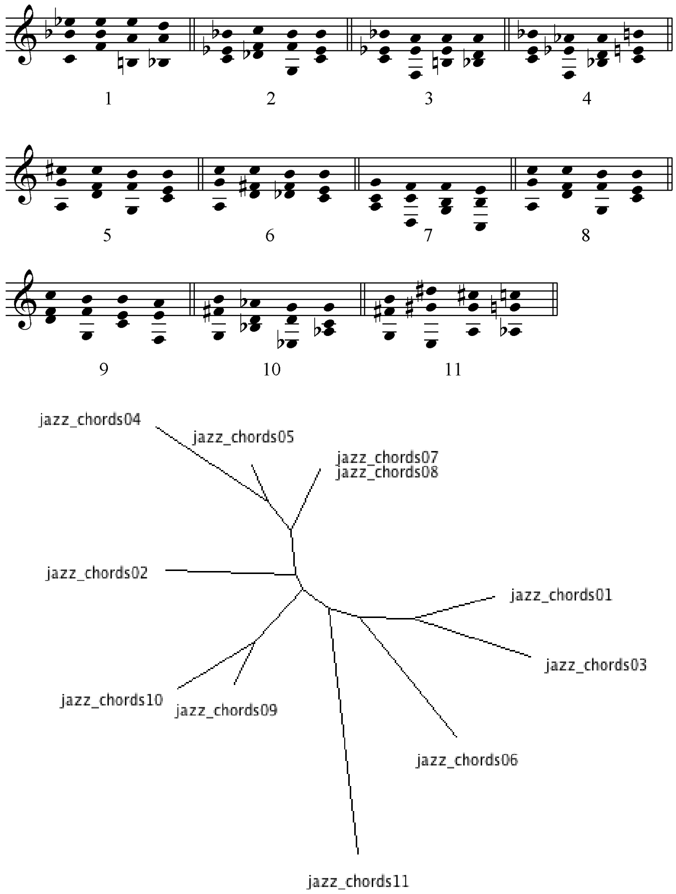

Figure 4 The jazz chord progressions are organised into a phylogenetic tree that illustrates their similarities and grouping based on their DIC vector distance. See text for more details.

V. CONCLUSIONS – FUTURE WORK

In this paper a novel chord transition representation has been proposed, wherein a harmonic transition between two chords can be represented as a Directional Interval Class (DIC) vector. This representation may be useful for practical computational modelling tasks, and at the same time, may have interesting ramifications for understanding musical harmony per se. Potentially useful properties of this representation include: easy to compute, independent of root finding, independent of key finding, incorporates voice leading qualities, preserves chord transition asymmetry, transposition invariant, independent of chord type, idiom-independent, chords can be uniquely derived from vector (except for symmetric chords such as the diminished seventh).

It is suggested that the proposed representation of chord transitions (as directed intervallic content) is analogous to the

[2] T:Tonic, S:Subdominant, D: Dominant

way melodic transitions between notes are being represented as pitch intervals. This representation may afford cognitive relevance in the sense that, in some cases at least, listeners may abstract and categorise directly intervallic content of chord progressions (instead of applying more advanced processing that involves identifying chord types, chord roots, circle-of-fifth relations between roots, and key). It is also suggested that listeners may encode not the full DIC vector, but rather a summary of it. For instance, small intervals (0-2 semitones) may be considered easier to perceive/encode as they commonly belong to independent melodic streams/voices; the same may apply for rare intervals (for tonal music) such as the tritone (6 semitones). Such smaller vectors may be considered more economic and cognitively plausible, and may allow quite efficient harmonic processing as they embody an important part of interval transition information.

Further research is required to establish more reliable distance metrics and clustering processes, and further testing on larger chord progression ground truth data sets (from music theory or empirically-derived). It is hoped that the Directed Interval Class representation may prove to be simple and potentially powerful representation of elementary harmonic relations that may have interesting ramifications in understanding harmony, and, additionally, for practical applications that involve harmonic encoding and processing.

ACKNOWLEDGMENT

Firstly and above all, I would like to express my gratitude to Andreas Katsiavalos for implementing the DIC-based harmonic similarity program and running it on the two test datasets. I am also grateful to Monika Doerfler for interesting discussions and for assisting me in testing the proposed model. Finally, I would like to thank Matthieu Giraud for showing that the DIC vector is not always in a one-to-one relation with the two chords of a chord transition (exception for symmetric chords).

REFERENCES

Allali, J., Ferraro, P., Hanna, P., & Iliopoulos, C. (2007). Local transpositions in alignment of polyphonic musical sequences. *Lecture Notes in Computer Science,* Volume 4726, pp. 26–38, Springer.

Allali, J., Ferraro P., Hanna P., & Robine M. (2010). Polyphonic Alignment Algorithms for Symbolic Music Retrieval. *Lecture Notes in Computer Science*, 2010, Volume 5954, pp. 466-482, Springer.

de Haas, W. B., Wiering, F. and Veltkamp, R. C. (2011). A Geometrical Distance Measure for Determining the Similarity of Musical Harmony. Technical Report UU-CS-2011-015, Department of Information and Computing Sciences, Utrecht University, Utrecht, The Netherlands.

de Haas, W. B., Veltkamp, R. C., and Wiering, F. (2008). Tonal pitch step distance: A similarity measure for chord progressions. In *Proceedings of the 9th International Conference on Music Information Retrieval* (ISMIR), pp. 51-56.

Hanna, P., Robine, M., and Rocher, T. (2009). An alignment based system for chord sequence retrieval. In Proceedings of the 2009 Joint International Conference on Digital Libraries, pages 101-104. ACM, New York.

Huson, D. H. 1998. "*Splitstree*: A Program For Analyzing And Visualizing Evolutionary Data." *Bioinformatics* 14.10: 68–73.

Kuusi, T. (2005) Chord span and other chordal characteristics affecting connections between perceived closeness and set-class similarity. *Journal of New Music Research,* 34(3), 257-271.

Lerdahl, F. (2001). *Tonal Pitch Space.* Oxford University Press, Oxford.

Levitin D.J. and Rogers S.E. (2005). Absolute pitch: perception, coding, and controversies, *TRENDS in Cognitive Sciences* 9(1), pp. 26-33.

Paiement, J.-F., Eck, D., and Bengio, S. (2005). A probabilistic model for chord progressions. In *Proceedings of the 6th International Conference on Music Information Retrieval* (ISMIR), pp. 312-319, London.

Pickens, J. and Crawford, T. (2002). Harmonic models for polyphonic music retrieval. In *Proceedings of the 11th International Conference on Information and Knowledge Management*, pp. 430-437. ACM, New York.

Temperley, D. (2001). *The Cognition of Basic Musical Structures.* MIT Press, Cambridge, MA.

Toussaint, G. T., Campbell, M., & Brown, N. (2011) Computational Models of Symbolic Rhythm Similarity: Correlation with Human Judgments, *Analytical Approaches to World Music, 1(2).*