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Wagner in the Round: Using Interval Cycles to Model Chromatic Harmony

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ABSTRACT

A formal grouping model is used to model the experience of tonal attraction within chromatic music, i.e. its dynamic "ebb and flow". The model predicts the level of tonal attraction between temporally adjacent chords. The functional ambiguity of nineteenth-century chromatic harmony can be problematic: chromatic chords, unlike diatonic harmony, often have ill-defined roots, and thus their proper functions are difficult to establish. An important feature of the model, however, is that the key or tonal context of the music does not need to be specified. The model is based on the idea of 'interval cycle proximity' (ICP), a grouping mechanism hypothesized to contribute to the perception of tonal attraction. This paper illustrates the model with an analysis of the opening of Wagner's 'Tristan und Isolde', and shows that the model can predict the opening sequence of 'Tristan' in terms of tonal attraction without the chords needing to be functionally specified.

I. INTRODUCTION

This research uses a cognitive model (see Appendix), designed to predict listeners' experience of tonal attraction (the dynamic "ebb and flow" of tonal music). The research applies the model to a particular problem from music theory, namely the correct functional description of chromatic harmony – chromatic chords, unlike diatonic harmony, often have ill-defined roots, which means that their functions are difficult to establish. Wagner's 'Tristan und Isolde' is an example *par excellence* of this, illustrated by the long-running debate over the correct functional interpretation of the opera's opening chords (Lerdahl, 2001, pp183-6). But the problem is not merely theoretical. When we listen to functionally ambiguous harmony, as at the opening of 'Tristan', are our perceptions unified or split? Music theory's multiple functional descriptions of chromatic music suggest that the latter might be the case, that functionally ambiguous harmony should be perceived as "either this, that, or both". However, anecdotally at least, this is not that case: chromatic music is not experienced ambiguously, but holistically as a unified percept, in much the same way as diatonic music. This implies that there is a mismatch between the functional, theoretical system we use to describe chromatic harmony, which leads to ambiguity, and our perceptions, which are for the most part unified and unambiguous.

The research presented in this paper explores whether or not the functional analysis of chromatic harmony can be supplemented by a perceptual, cognitive approach and model.

II. MODEL

Woolhouse (2009) presented a formal, context-independent model that predicts the level of tonal attraction between temporally adjacent pitches and/or chords. The model is independent of the context in that the key or tonal context of the music is not specified in the calculation. The model is based on interval cycle proximity, a grouping process which is hypothesized to be responsible for the perception of tonal attraction (Woolhouse & Cross, 2010).

For a given chord the model can output a tonal attraction 'profile', a graph in which the tonal attraction of the chord is displayed in relation to the pitches of a scale or the chromatic octave. Figure 1 shows the tonal attraction profile produced by a G^7 chord in relation to the pitch classes of the C harmonic minor scale. Briefly stated, the model predicts that a chord containing the notes {G, B, D, F}, i.e. a dominant-seventh chord, will be strongly attracted to {C}, and weakly attracted to {F} and {B}, with the other pitches of C harmonic minor somewhere in between.

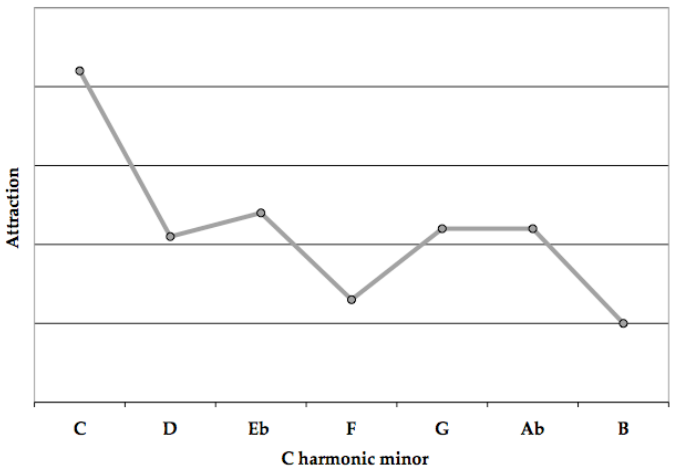

Figure 1. The model's tonal attraction profile for each pitch of the C harmonic minor scale following a G^7 chord {G, B, D, F}.

Woolhouse (2009) tested the model by obtaining listeners' subjective ratings of tonal attraction for various individual chords followed by individual pitches (probe tones); a high level of correlation was found between the model and listeners' responses.

Among the chords tested was the half-diminished seventh {C, Eb, Gb, Bb}. Figure 2 shows the model's tonal attraction profile and the listeners' rating data for this chord in relation to the pitch classes of the chromatic octave. The correlation coefficient between the model and data is highly significant: $r(10) = 0.89$; $p < 0.005$. The model correctly predicted the three main peaks in the data on Db, F and B (although not quite in the correct order). The two main troughs in the data on C and Gb were also correctly predicted. Less successfully modelled, however, were the data for D and E, which were higher than expected, and the data for pitches Eb and Ab, which were lower than expected. Despite these discrepancies, the fit of the data to the model was very good.

Half-diminished seventh chords are enharmonic versions of the Tristan chord, {D#, F, G#, B}, i.e. a Tristan chord can be created by re-spelling the notes of the half-diminished chord {Eb, F, Ab, Cb} (in this case, {F} is the functional root of the half-diminished seventh). Given that the model predicted listeners' experience of tonal attraction for a half-diminished chord when presented in isolation, it is reasonable to ask whether or not the model can predict the harmonic behaviour of the Tristan chord at the opening of Wagner's opera 'Tristan und Isolde' (1855-59).

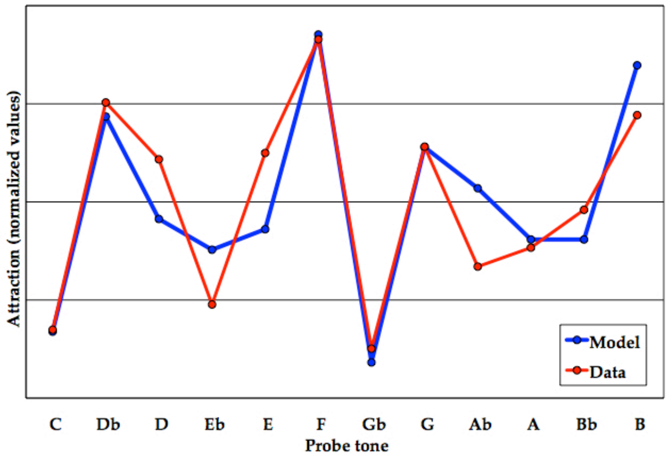

Figure 2. Listeners' responses (red line) and the model's attraction profile (blue line) for each pitch of the chromatic octave following a half-diminished seventh chord {C, Eb, Gb, Bb}: r(10) = 0.89; p < 0.005.

III. THE TRISTAN CHORD

The first chord in 'Tristan und Isolde' is referred to as the Tristan chord because of its appearance in the *Sehnsucht*, "longing", *leitmotive* (the "longing" refers to Tristan's unrequited love for Isolde). As mentioned above, the Tristan chord, indicated by the asterisk in Figure 3, is the subject of much theoretical speculation regarding its correct functional description. The argument largely centres on whether or not the G# in bar 2 is interpreted as an appoggiatura. If the G# is not an appoggiatura, then * is a chord in its own right, whose functional root is G# (see upper analysis in A minor, Figure 3); if, however, the G# is an appoggiatura resolving to A, then * is a latent French sixth (realized at O) whose functional root is B (see lower analysis in A minor, Figure 3).

Figure 3. Richard Wagner, 'Tristan und Isolde' (1857-9); bars 1-3. The progression is analysed in A minor.

Mitchell (1967) is among those who view the Tristan chord as being a distinct chord in its own right, while those who see it as belonging to the French sixth include Kurth (1920), Piston (1941) and Goldman (1965). Lerdahl (2001) accepts both interpretations as plausible; Wason (1985) and Nattiez (1990) discuss the interpretation of the chord from a comparative perspective.

One possible reason for the longevity of the debate surrounding the Tristan chord (the controversy began shortly after the opera's first performance; see Magee, 2000) is that the nomenclature of functional tonality is more suited to harmonically unambiguous situations; that is, music in a clearly defined key in which the vertical elements are to some degree homophonic. From the mid-nineteenth century onwards, however, harmony became increasingly ambiguous with respect to its functionality. The New Grove Dictionary of Music (1980) summarizes the situation in the following way:

> [The Tristan chord] played an important role in the last developments of chromatic harmony in the late nineteenth century and the early twentieth, and seems to have been crucial to the *limitation of the applicability of functional theory to harmonic analysis"* (my emphasis).

This notion was also expressed by Erickson (1975) who wrote, "The Tristan chord is, among other things, an identifiable sound, an entity beyond its functional qualities in a tonal organization" (p.18).

A potential benefit of the interval cycle proximity model with respect to chromatic harmony, and with respect to the ambiguity concerning the Tristan chord in particular, is that tonal attractions values are calculated without reference to functionality, i.e. context independent. In order to address the issue of whether or not the model could be used to predict the opening of 'Tristan und Isolde', the model's tonal attraction profile for the half-diminished seventh chord (Figure 2, grey line) was transposed so that it matched the pitches of the Tristan chord at the start of the opera, {D#, F, G#, B}; see Figure 4.

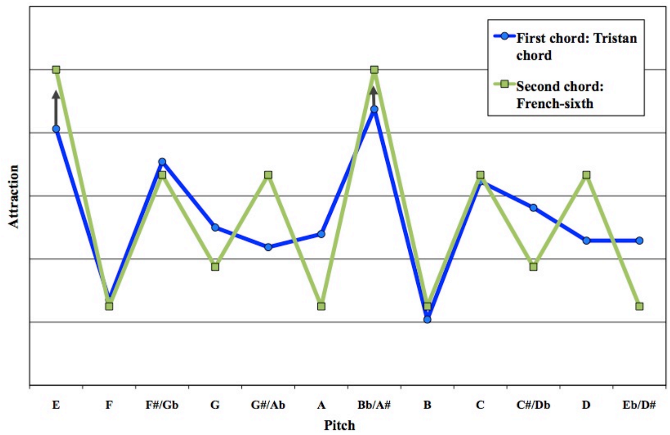

Figure 4. The model's tonal attraction profiles of the first two chords of Wagner's opera 'Tristan und Isolde', calculated in relation to the pitch classes of the chromatic octave. The tonal attraction profile of the Tristan chord {D#, F, G#, B} is blue (* in Figure 3); the tonal attraction profile of the French sixth {D#, F, A, B} is green (O in Figure 3).

Also in Figure 4 is the model's tonal attraction profile for the second chord in 'Tristan und Isolde', the French sixth {D#, F, A, B} (bar 2, sixth quaver beat). Figure 4 therefore presents the tonal attraction profiles of the first two chords of the opera, the Tristan chord (*), followed by the French sixth (O).

From the tonal attraction profiles in Figure 4 the following predictions can be made regarding the behaviour of the Tristan chord and French sixth. First, the Tristan chord should be most strongly attracted towards {E} and/or {A#}; and second, the level of attraction to {E} and/or {A#} should be strengthened by the French sixth (see Figure 4, vertical arrows). This prediction is strikingly similar to the music of the opening phrase: despite the lugubrious *langsam und schmachtend* tempo marking (slowly and languishing), the Tristan chord creates a strong sense of momentum and imminent resolution, which is strengthened by the French sixth, and then resolved, initially onto an {E} in the bass and an {A#} in the upper voice at the beginning of bar 3, and finally onto an E^7 chord on the second quaver beat of bar 3 (see Figure 3). In other words, the model predicts the movement/resolution of the chords in bar 2 to the most salient notes (Huron, 1989; Huron & Fantini, 1989) at the beginning of bar 3: the outer notes {E} and {A#}, and the resolution of the phrase onto a chord with the root-note {E}.

IV. CONCLUSION

The predictive, analytical approach outlined above was carried out with neither the Tristan chord (* in Figure 3) or the subsequent French sixth (O in Figure 3) being described functionally. The method employed a model in which tonal attraction, the dynamic "ebb and flow" of the music, was determined primarily by interval cycles, values that are invariant with respect to how the chords that constitute them are functionally described or enharmonically spelt. And in this respect, I propose that the model reflects the manner in which this music is perceived; that is, not as a functionally split or divided musical experience, as a traditional music-theoretic description might indicate, but as a unified percept in which the tones of the music have propensities that pull, push and attract one another in a coherent and principled manner.

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APPENDIX: MODEL

A. Music

Given two successive chords (a chord is defined as a simultaneity containing one or more notes): past chord, X, and present chord, Y

$X = \{x_1, x_2, \ldots, x|X|\}$ $\qquad x_1 < x_2 < \ldots < x|X|$

$Y = \{y_1, y_2, \ldots, y|Y|\}$ $\qquad y_1 < y_2 < \ldots < y|Y|$

where |X| = size of set X, |Y| = size of set Y, and where x_i and y_j are defined with reference to $C_4 = 60$.

B. Pitch Distance

Form matrix PD where

$$PD_{ij} = |y_j - x_i| \qquad i = 1, 2, \ldots, |X| \quad j = 1, 2, \ldots, |Y|$$

C. Interval Cycles

Form matrix IC where

$$IC_{ij} = \frac{12}{hcf(PD_{ij}, 12)} \quad \text{for } PD_{ij} \neq 0$$

$IC_{ij} = 1$ if $PD_{ij} = 0$

where *hcf*(a, b) is the highest common factor of a and b, and a, b are whole numbers.

D. Voice Leading

Define a further matrix VL such that

$$VL_{ij} = \frac{\alpha}{PD_{ij} + \alpha} \quad \text{for some } \alpha > 0$$

E. Interval Cycles & Voice Leading

Using entry-wise multiplication (array multiplication), combine matrix VL with matrix IC to give matrix ICVL, defined by

$ICVL_{ij} = VL_{ij}\, IC_{ij}$

F. Root Salience

Form matrix RS1 where

$$RS1_{ij} = 1 \qquad i = 1, 2, \ldots, |X| \qquad j = 1, 2, \ldots, |Y|$$

If either chord has an identifiable root, let the row corresponding to the past root be the m^{th} row and the column corresponding to the present root be the n^{th} column.

Form matrix RS2 where

$$RS2_{ij} = \begin{cases} RS1_{ij} \text{ if } i \neq m \\ RS1_{ij} \text{ if } j \neq n \\ \beta \times RS1_{ij} \text{ if } i = m \text{ for some } \beta > 1 \\ \gamma \times RS1_{ij} \text{ if } j = n \text{ for some } \gamma > \beta \end{cases}$$

such that root intersection entry $RS2_{mn} = \beta \times \gamma \times RS1_{mn}$. If neither chord has an identifiable root, form matrix RS2 where

$$RS2_{ij} = RS1_{ij}$$

Form matrix RS3 where

$$RS3_{ij} = \frac{RS2_{ij}}{\sum_{i=1}^{|X|}\sum_{j=1}^{|Y|} RS2_{ij}}$$

G. Consonance & Dissonance

If chords X and Y are both sensory consonances, or both are sensory dissonances, form matrix CD where

$$CD_{ij} = RS3_{ij}$$

If chord X (past) is dissonant and chord Y (present) is consonant, form matrix CD where

$$CD_{ij} = (1 + \delta) \times RS3_{ij} \text{ for some } \delta > 0$$

If chord X (past) is consonant and chord Y (present) is dissonant, form matrix CD where

$$CD_{ij} = (1 - \delta) \times RS3_{ij} \text{ for some } \delta > 0$$

H. Tonal Attraction

Using entry-wise multiplication (array multiplication), combine matrix ICVL with matrix CD to give matrix TA, defined by

$$TA_{ij} = ICVL_{ij}\, CD_{ij}$$

Sum the entries in matrix TA to produce the overall attraction value, A, where

$$A = \frac{\sum_{i=1}^{|X|}\sum_{j=1}^{|Y|} TA_{ij}}{12}$$